Cosmological Daemon

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Проблема необратимости в классических и квантовых динамических системах

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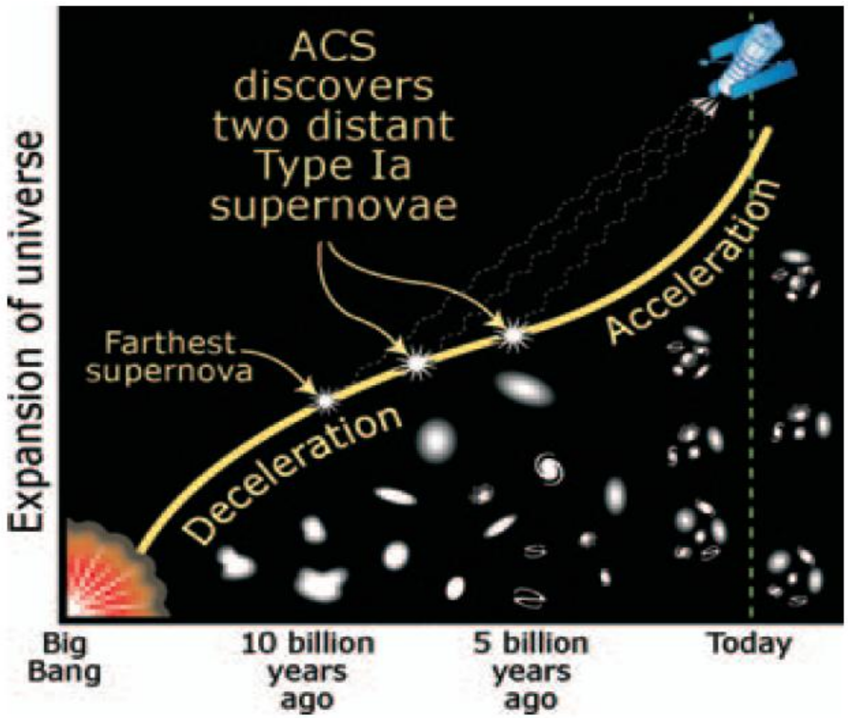
Subject of the talk:

Early cosmology (14 Billions years before)

From Big Bang to Life on Earth

Brief History of the Universe



First Big Bang.

Then inflation.

General Relativity

- **General Relativity and Friedmann Cosmology.**

- **Dynamics of Spacetime:**

$$(M, g)$$

$$M \quad - \quad 4-\text{dim manifold},$$

$$g = (g_{\mu\nu}) \quad - \quad \text{metric}$$

Cosmological principle:

Homogeneous

Isotropic

Friedmann metric

$$ds^2 = -dt^2 + a^2(t)dx^2$$

FRW cosmology with one (local) scalar field

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_p^2}{2}R + \Phi\Box\Phi - V(\Phi)\right\}$$

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu \qquad m_p^2 = \frac{1}{8\pi G}$$

FRW:

$$ds^2 = -dt^2 + a^2(t)dx^2 \qquad \Box = -\partial_t^{\,2} - 3H\partial_t$$

$$H = \frac{\dot{a}}{a}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'_\phi$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Model

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_p^2}{2} R + \Phi F(\Box)\Phi - V(\Phi) \right\}$$

$$m_p^2 = \frac{1}{8\pi G}$$

Superstring inspired

$$F(\Box) = e^{\tau \Box}(-\Box - \mu^2),$$

τ *is a parameter from SFT*

$$V(\Phi) = \frac{\varepsilon}{4}\Phi^4$$

Nonlocal Models in Cosmology

I. Nonlocality in Matter (*mainly string motivated*)

$$\int d^4x\sqrt{-g}\left(\frac{1}{g_4^2}\left(\frac{1}{2}\Phi F\left(\frac{\Box_g}{M_s^2}\right)\Phi - V(\Phi)\right) + \frac{M_p^2}{2}R\right)$$

- **Later cosmology**
  w<-1

- **I.A., astro-ph/0410443**
  I.A., L.Joukovkaya, JHEP,05109 (2005) 087
  I.A., A.Koshelev, JHEP, 07022 (2007) 041
  L.Joukovskaya, PR D76(2007) 105007; JHEP (2009)

- **Inflation**
  steep potential,
  non-gaussianity

G. Calcagni, M.Montobbio,G.Nardelli,0705.3043; 0712.2237; Calcagni, Nardelli, 0904.4245; IA, N.Bulatov, L.Joukovskaya, S.Vernov, 0911.5105; PR(2009)

- **Bouncing solutions**

I.A., L.Joukovskaya, S.Vernov, JHEP 0707 (2007) 087
Nunes, Mulryne, 0810.5471;
N. Barnaby, T. Biswas, J.M. Cline, hep-th/0612230,
J.Lidsey, hep-th/0703007;

Nonlocal Models in Cosmology

II. Nonlocality in Gravity

$$\int d^4x\sqrt{-g}\left(\frac{1}{g_4^2}\left(\frac{1}{2M_s^2}\Phi\Box\Phi-V(\Phi)\right)+\frac{M_p^2}{2}G^{\mu\nu}F\left(\frac{\Box}{M},\ldots\right)R_{\mu\nu}\right)$$

Arkani-Hamed at al hep-th/0209227; Khoury, hep-th/0612052
T.Biswas, A.Mazumdar, W.Siegel hep-th/0508194 ,
G.Dvali, S. Hofmann, J Khoury, hep-th/0703027,
S.Deser, R.Woodard, arXiv:0706.2151S. **UV - completion**

Main messages

- Eqs with infinite number of derivatives on the half line

⇓

- in additional to the usual initial data a new arbitrary function

I. A., I.Volovich, JHEP 08 (2011)102, arXiv:1103.0273

Outlook

i) How to understand $F(\Box) = e^{\tau\Box}(-\Box - \mu^2),$

ii) Why half-axis ?

iii) Possible applications

Nonlocal operator of SFT-type

$$F(\Box) = e^{\tau\Box}(-\Box + \mu^2), \quad \Box = \partial^2_{tt} - \partial^2_{x_i x_i}$$

On space of homogeneous functions

$$F(\Box) = F(\partial^2_t) = e^{\tau\,\partial^2_t}(-\partial^2_t + \mu^2),$$

$e^{\tau\,\partial^2_t}$ **as solution of the diffusion equation**

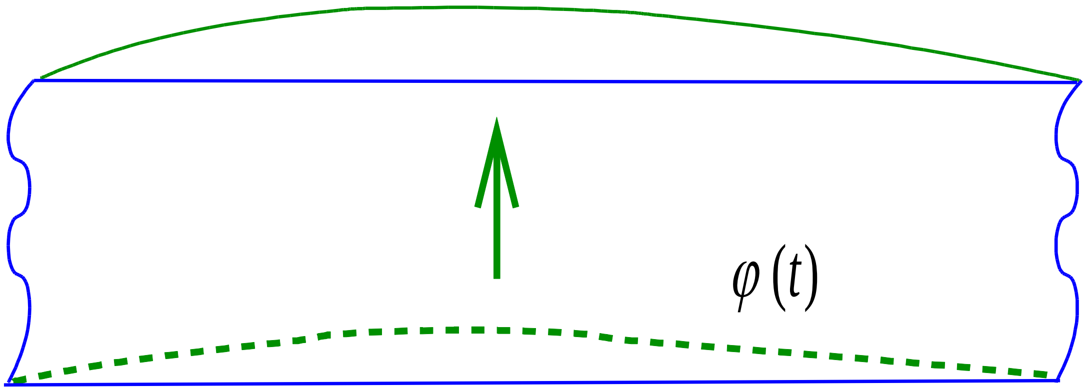
$$\Psi(\tau, t) = e^{\tau\partial^2_t}\varphi(t) \iff \begin{cases} (\partial_\tau - \partial^2_t)\Psi(t,\tau) = 0, \\ \Psi(t,\tau)|_{\tau=0} = \varphi(t). \end{cases}$$

The Heat Equation on the Whole Line

$$(\partial_\tau - \partial_t^2)\Psi(t,\tau) = 0,$$

$$\Psi(t,\tau)|_{\tau=0} = \varphi(t).$$

$$\Psi(t,\tau) \equiv e^{\tau\partial_t^2}\varphi(t) = \mathcal{K}[\varphi](t)$$



$$\mathcal{K}[\varphi](t) \equiv \frac{1}{\sqrt{4\pi\tau}} \int\limits_{-\infty}^{\infty} \varphi(t')\, e^{-\frac{(t-t')^2}{4\tau}}\, dt'$$

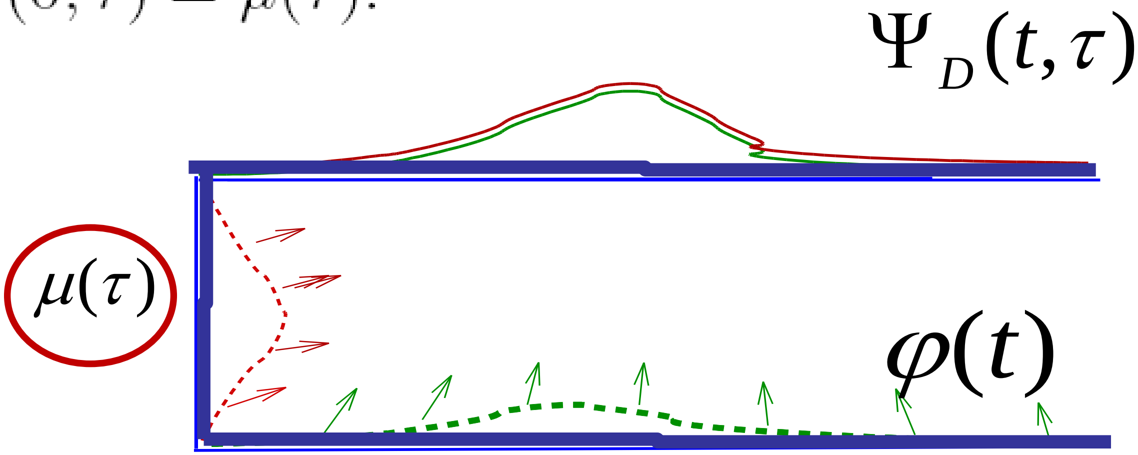
Cosmological Singularity

- Classical versions of the Friedmann Big Bang cosmological models of the universe contain a singularity at the start of time.

Proposal: restrict ourself by considering the time variable t running over the half-line with regular boundary conditions at $t = 0$.

The heat equation of the half-line

$$\begin{cases} \frac{\partial}{\partial \tau}\Psi_D(t,\tau) = \frac{\partial^2}{\partial t^2}\Psi_D(t,\tau), \quad t>0, \quad \tau>0, \\ \Psi_D(t,0) = \varphi(t), \\ \Psi_D(0,\tau) = \mu(\tau). \end{cases}$$



$$\Psi_D(t,\tau) = \frac{1}{\sqrt{4\pi\tau}}\int_0^\infty \varphi(t')\,[e^{-\frac{(t-t')^2}{4\tau}} - e^{-\frac{(t+t')^2}{4\tau}}]dt'$$

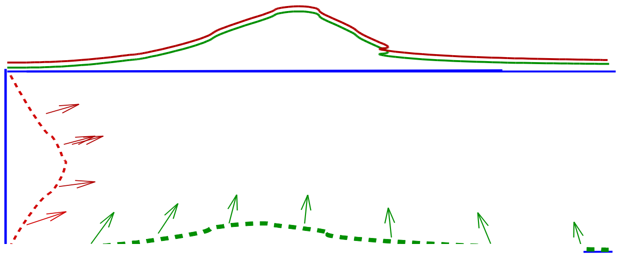
$$+\frac{t}{\sqrt{4\pi}}\int_0^\tau \frac{\mu(\tau')}{(\tau-\tau')^{3/2}}e^{-\frac{t^2}{4(\tau-\tau')}}\,d\tau' \equiv J(t)$$

Solutions to Linearized Nonlocal Equation on the Half-line

- Dirichlet's daemon without source

$$e^{\tau\partial_t^2}_{D,0}\Phi(t) = m^2\Phi(t), \quad t > 0.$$

$\mu = 0$



$$\frac{1}{2\sqrt{\tau\pi}}\int_0^\infty dt'\,[e^{-\frac{(t-t')^2}{4\tau}} - e^{-\frac{(t+t')^2}{4\tau}}]\,\Phi(t') = m^2\Phi(t), \quad t > 0.$$

$$\Phi_D(t) = \sum_n \left(\frac{1}{2}B_n(e^{\alpha_n t} - e^{-\alpha_n t}) + \frac{1}{2}B_n^*(e^{\alpha_n^* t} - e^{-\alpha_n^* t})\right)$$

$$e^{\tau\alpha_n^2} = m^2 \qquad \alpha_n = \pm\sqrt{\frac{\ln m^2 + 2\pi i n}{\tau}}, n = 0, \pm 1, \pm 2, \ldots$$

Semantic remarks

J(x) – Daemon external source.

- **According to Plato: daemons are good or benevolent "supernatural beings between mortals and gods"**
- **Judeo-Christian usage of demon: a malignant spirit.**
- **Socrates‘ daimon is analogous to the guardian angel.**

Profit from the Daemon external source

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} R + \frac{1}{2} \phi \Box \phi - U(\phi) + J\phi \right\},$$

J(x) – **Daemon external source**

$$\epsilon = \frac{M_P^2}{2} \left(\frac{U'(\phi) - J}{U(\phi) - J\phi} \right)^2$$

$$U(\phi) = m^2\phi^2/2 \,;\, \epsilon = \frac{M_P^2}{2\phi^2} \left(\frac{m^2\phi - J}{\frac{m^2}{2}\phi - J} \right)^2 = \frac{M_P^2}{2\phi^2} \frac{\delta^2}{(\frac{m^2}{2}\phi - \delta)^2}$$

$$\delta = m^2\phi - J$$

Summary

- **Eqs with infinite number of derivatives on the half line lead to a new arbitrary function**

Non-locality. Definition of $F(\Box)$

$F(\Box)$ **with the Fourier transform**

$$F(\Box)\phi(x) = \frac{1}{(2\pi)^n}\int F(-\xi^2)\tilde{\phi}(\xi)e^{-ix\xi}d^n\xi$$

where

$$\tilde{\phi}(\xi) = \int \phi(x)e^{i\xi x}d^n x$$

This definition is natural is SFT, where all expressions came from calculations in the momentum space

$$\Box \Rightarrow -\partial_t^2 \qquad -\infty \le t \le \infty$$

Non-locality. Definition of $F(\Box)$?

$F(\partial_t^2)$ **as a symbol with the Laplace transformation**

$$F(\partial_t^2)\varphi(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s^2)\tilde{\varphi}(s)e^{ts}ds$$

where

$$\tilde{\varphi}(\xi) = \int_0^{\infty} \varphi(t)e^{-st}dt$$

Definition depends on "c"

Non-locality in level truncation

With the Laplace transformation with a closed contour

$$f(\partial_t)\phi(t) = \frac{1}{2\pi i}\oint_C ds\, e^{st}\left[f(s)\tilde{\phi}(s) - \sum_{n=0}^{\infty}\sum_{i=1}^{n}\frac{f^{(n)}(0)}{n!}d_j s^{n-j}\right]$$

where

$$\tilde{\varphi}(\xi) = \int_0^{\infty}\varphi(t)e^{-st}dt \qquad 0 \le t \le \infty$$

$$d_j = \partial_t^{(j-1)}\phi(0)$$

Suitable for the Cauchy problem

N.Barnaby, N.Kamran to study cosmological perturbations, arXiv:0712.2237;arXiv:0809.4513 (previous works by R.Woodard and coauthors)