# Non-Newtonian Mechanics and Irreversibility Problem

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# PLAN

**Time Irreversibility Problem**

**Non-Newtonian Classical Mechanics**

**Bogolyubov type Equations**

**for Two Particles**

**Functional Probabilistic General Relativity**

I. V. *Found. Phys.* (2011).

- I.V. Theor. Math. Phys (2010)

# Newton Equation

$$m\frac{d^2}{dt^2}x = F(x),$$

$$x = x(t),$$

$$p = m\frac{dx}{dt},$$

$$(M = R^{2n}, \varphi_t)$$

**Phase space (q,p), Hamilton dynamical flow**

# Why Newton`s mechanics can not be true?

- Newton`s equations of motions use real numbers while one can observe only rationals. (s.i.)

- Classical uncertainty relations

- Time irreversibility problem

- Singularities in general relativity

- **Try to solve these problems by developing a new, non-Newtonian mechanics.**

# Classical Uncertainty Relations

$$\Delta q > 0, \quad \Delta p > 0$$

$$\boldsymbol{\Delta t > 0}$$

# Newton`s Classical Mechanics

**Motion of a point body is described by the trajectory in the phase space.**

**Solutions of the equations of Newton or Hamilton.**

**<u>Idealization: Arbitrary real numbers—non observable.</u>**

- **<u>Newton`s mechanics deals with non-observable (non-physical) quantities.</u>**

# Real Numbers

- **A real number is an infinite series, which is unphysical:**

$$t = \sum_n a_n \frac{1}{10^n}, \; a_n = 0, 1, \ldots, 9.$$

$$m \frac{d^2}{dt^2} x(t) = F$$

# Rational numbers. P-Adic numbers

V.S. Vladimirov, I.V.V., E.I. Zelenov,

B. Dragovich, A.Yu. Khrennikov, S.V. Kozyrev, V.Avetisov, A.Bikulov,

A.P.Zubarev,…Witten, Freund, Frampton,…

- Journal: “p-Adic Numbers,…”

# Time Irreversibility Problem

**The time irreversibility problem is the problem of how to explain the irreversible behaviour of macroscopic systems from the time-symmetric microscopic laws:** $t \to -t$

**Newton, Schrodinger Eqs –- reversible**

**Navier-Stokes, Boltzmann, diffusion,**
**Entropy increasing --- irreversible**

$$\frac{\partial u}{\partial t} = \Delta u.$$

# Time Irreversibility Problem

**Boltzmann, Maxwell, Poincar´e, Bogolyubov, Kolmogorov, von Neumann, Landau, Prigogine, Feynman, Kozlov,…**

**Poincar´e,  Landau, Prigogine, Ginzburg, Feynman: Problem is open.**
**We will never solve it (Poincare)**
**Quantum measurement? (Landau)**

**Lebowitz, Goldstein, Bricmont:**
**Problem was solved by Boltzmann**

# Loschmidt paradox

- From the symmetry of the Newton equations upon the reverse of time it follows that to every motion of the system on the trajectory towards the equilibrium state one can put into correspondence the motion out of the equilibrium state if we reverse the velocities at some time moment.

- Such a motion is in contradiction with the tendency of the system to go to the equilibrium state and with the law of increasing of entropy.

# Poincar´e–Zermelo paradox

- **Poincar´e recurrence theorem:**
  **a trajectory of a bounded isolated mechanical system will be many times come to a very small neighborhood of an initial point.**

- **Contradiction with the motion to the equilibrium state.**

# Boltzmann`s answers to:

Loschmidt: statistical viewpoint

Poincare—Zermelo: extremely long Poincare recurrence time

Coarse graining

- Not convincing…

# Ergodicity

- **Boltzmann, Poincare, Hopf, Kolmogorov, Anosov, Arnold, Sinai,…:**

- **Ergodicity, mixing,… for various important deterministic mechanical and geometrical dynamical systems**

# Bogolyubov method

1. Newton to Liouville Eq.
   Bogolyubov (BBGKI) hierarchy
2. Thermodynamic limit (infinite number of particles)

3. The condition of weakening of initial correlations between particles in the distant past

4. Functional conjecture
5. Expansion in powers of density

*Divergences.*

*A.S.Trushechkin.*

# Weak Limit for Gas (Fluid)

**A. Poincare, V.V. Kozlov, D.V.Treschev,…**

**Free motion of gas particles in a box with reflecting walls**

**Irreversible diffusion**

**Other models**

# Stochastic Limit

and Irreversibility

**L.Accardi, I.V., Yu.G.Lu, S.V.Kozyrev,**

**A.N. Pechen,…**

**Noncommutative probability**

**Quantum Field Theory, Quantum Optics**

# Functional Formulation of Classical Mechanics

**Usual approaches to the irreversibility problem:**

**Start from Newton Eq.  Gas of particles**

**Derive Boltzmann Eq.**

**This talk: Irreversibility for one particle**

**Modification of the Newton approach to Classical mechanics: Functional formulation**

**We attempt the following solution of the irreversibility problem:**
**a formulation of microscopic dynamics which is irreversible in time: Non-Newtonian Functional Approach.**

# Functional formulation of classical mechanics

- Here the physical meaning is attributed not to an individual trajectory but only to a bunch of trajectories or to the distribution function on the phase space. The fundamental equation of the microscopic dynamics in the proposed "functional" approach is not the Newton equation but the Liouville or Fokker-Planck-Kolmogorov (Langevin, Smoluchowski) equation for the distribution function of the single particle.

# States and Observables in Functional Classical Mechanics

$$(q, p) \in \mathbb{R}^2 \qquad \text{(phase space)},$$

$$\rho = \rho(q, p, t) \qquad \text{state of a classical particle}$$

$$\rho \geq 0, \quad \int_{\mathbb{R}^2} \rho(q, p, t) dq dp = 1, \; t \in \mathbb{R}\,.$$

# States and Observables in Functional Classical Mechanics

$$\overline{f}(t) = \int f(q,p)\rho(q,p,t)dqdp\,.$$

$f(q,p)$ is a function

**Not a generalized function**

# Fundamental Equation in Functional Classical Mechanics

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V(q)}{\partial q}\frac{\partial \rho}{\partial p}.$$

**Looks like the Liouville equation which is used in statistical physics**
**to describe a gas of particles but here we use it to describe a <u>single particle</u>.(moon,…)**

**Instead of Newton equation. <u>No trajectories!</u>**

# Cauchy Problem for Free Particle

$$\rho\big|_{t=0} = \rho_0(q,p)\,.$$

$$\rho_0(q,p) = \frac{1}{\pi ab} e^{-\frac{(q-q_0)^2}{a^2}} e^{-\frac{(p-p_0)^2}{b^2}}\,.$$

# Average Value and Dispersion

$$\overline{q} = \int q\rho_0(q,p)dqdp = q_0\,, \quad \overline{p} = \int p\rho_0(q,p)dqdp = p_0\,,$$

$$\Delta q^2 = \overline{(q-\overline{q})^2} = \frac{1}{2}a^2, \quad \Delta p^2 = \overline{(p-\overline{p})^2} = \frac{1}{2}b^2\,.$$

# Free Motion

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q}$$

$$\rho(q, p, t) = \rho_0(q - \frac{p}{m}t, p)\,.$$

# Delocalization

$$\rho_c(q,t) = \int \rho(q,p,t)dp = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{b^2t^2}{m^2}}} \exp\{-\frac{(q - q_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{b^2t^2}{m^2})}\}$$

$$\Delta q^2(t) = \frac{1}{2}(a^2 + \frac{b^2t^2}{m^2})$$

# Newton`s Equation for Average

$$\overline{q}(t) = \int q\rho_c(q,t)dq = q_0 + \frac{p_0}{m}t\,, \quad \overline{p}(t) = \int p\rho_m(p,t)dp = p_0\,.$$

$$\frac{d^2}{dt^2}\overline{q}(t) = 0\,,$$

# Comparison with Quantum Mechanics

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

$$\rho_q(x,t) = |\psi(x,t)|^2 = \frac{1}{\sqrt{\pi}\sqrt{a^2+\frac{\hbar^2t^2}{a^2m^2}}}\exp\{-\frac{(x-x_0-\frac{p_0}{m}t)^2}{(a^2+\frac{\hbar^2t^2}{a^2m^2})}\}$$

$$a^2b^2 = \hbar^2$$

$$W(q,p,t) = \rho(q,p,t)$$

# Liouville and Newton. Characteristics

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{k} \frac{\partial}{\partial x^i}(\rho v^i) = 0$$

$$\dot{x} = v(x)$$

$$\rho\big|_{t=0} = \rho_0(x)$$

$$\rho(x, t) = \rho_0(\varphi_{-t}(x))$$

# Corrections to Newton`s Equations
# Non-Newtonian Mechanics

$$\rho_0(q,p) = \delta_\epsilon(q-q_0)\delta_\epsilon(p-p_0)$$

$$\delta_\epsilon(q) = \frac{1}{\sqrt{\pi}\epsilon} e^{-q^2/\epsilon^2} \ ,$$

**Proposition 1.** Newton's Equations

$$\lim_{\epsilon \longrightarrow 0} \int f(q,p)\rho(q,p,t)dqdp = f(\varphi_t(q_0,p_0)) \ .$$

# Corrections to Newton`s Equations

$$\frac{\partial \rho}{\partial t} = -p\frac{\partial \rho}{\partial q} + \lambda q^2 \frac{\partial \rho}{\partial p}$$

$$\dot{p}(t) + \lambda q(t)^2 = 0\,, \;\; \dot{q}(t) = p(t)\,.$$

# Corrections to Newton`s Equations

**Proposition 2.**

$$< q(t) >= q_{\mathrm{Newton}}(t) - \frac{\lambda}{4}\epsilon^2 t^2$$

$$q_{\mathrm{Newton}}(t) = q_0 + p_0 t - \frac{\lambda}{2} q_0^2 t^2$$

$-\frac{\lambda}{4}\epsilon^2 t^2$ is the correction to the Newton solution

- 
- The Newton equation in this approach appears as an approximate equation describing the dynamics of the expected value of the position and momenta for not too large time intervals.
- Corrections to the Newton equation are computed.
- ________________________________
- ________________________________

?

# Stability of the Solar System?

- **Kepler, Newton, Laplace, Poincare,…,Kolmogorov, Arnold,…---stability?**

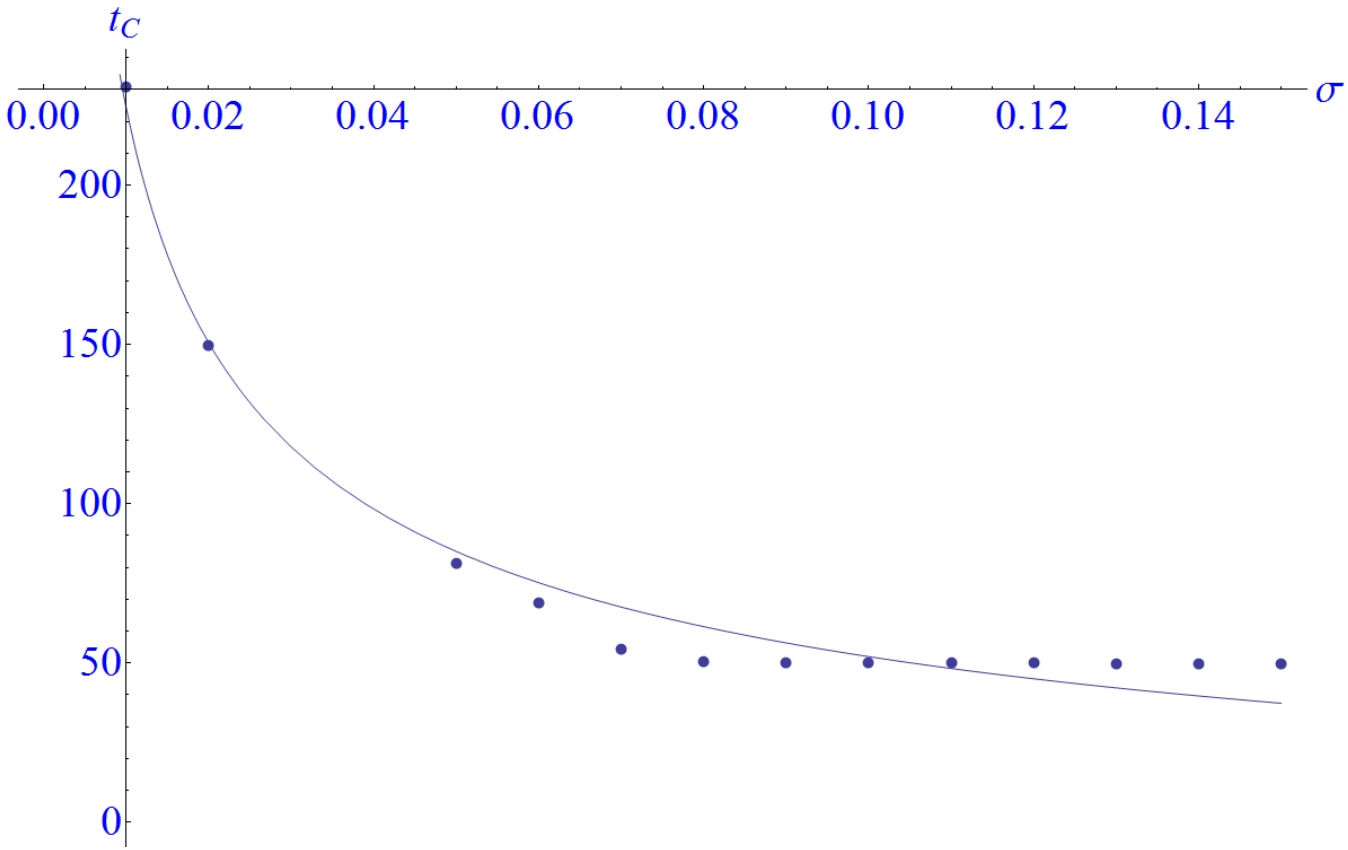
- **Solar System is unstable, chaotic (J.Laskar)**
- Laskar's equations had some 150,000 terms. Laskar's work showed that the Earth's orbit (as well as the orbits of all the inner planets) is chaotic and that an error as small as 15 metres in measuring the position of the Earth today would make it impossible to predict where the Earth would be in its orbit in just over 100 million years' time.

# Corrections

$$m\frac{d^2}{dt^2} < q(t) > = < F(q)(t) >$$

**E. Piskovsky,**

**A. Mikhailov**




$$|\langle q\rangle(t_C,\sigma) - q_{KB}(t_C,b)| = 0.1q_{(KB)}(0,\sigma)$$

$$t_C = O(1/\sqrt{\sigma})$$

- **Newton`s approach: Empty space (vacuum) and point particles.**
- **Reductionism:  For physics, biology economy, politics (freedom, liberty,…)**

- **This approach: No empty space. Probability distribution. Collective phenomena. Subjective.**

# Fokker-Planck-Kolmogorov versus Newton

$$\frac{\partial f}{\partial t} - \{H, f\} = \sigma \frac{\partial^2 f}{\partial p^2} + \gamma \frac{\partial (pf)}{\partial p}$$

# Boltzmann and Bogolyubov Equations

- **A method for obtaining kinetic equations from the Newton equations of mechanics was proposed by Bogoliubov. This method has the following basic stages:**
- **Liouville equation for the distribution function of particles in a finite volume, derive a chain of equations for the distribution functions,**
- **pass to the infinite-volume, infinite number of particles limit,**
- **postulate that the initial correlations between the particles were weaker in the remote past,**
- **introduce the hypothesis that all many-particle distribution functions depend on time only via the one-particle distribution function, and use the formal expansion in power series in the density.**

- **Non-Newtonian Functional Mechanics:**
- **Finite volume. Two particles.**

- **A.Trushechkin.**

# Liouville equation for two particles

$$\rho = \rho(x_1, x_2, t)$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_1}\frac{\partial \rho}{\partial p_1} - \frac{p_1}{m}\frac{\partial \rho}{\partial q_1} + \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_2}\frac{\partial \rho}{\partial p_2} - \frac{p_2}{m}\frac{\partial \rho}{\partial q_2},$$

# Two particles in finite volume

$$f_1(x_1, t) = V \int_{\Omega_V} \rho(x_1, x_2, t)\, dx_2, \qquad f_2(x_1, x_2, t) = V^2 \rho(x_1, x_2, t),$$

$$f_2(x_1, x_2, t_0) = f_1(x_1, t_0) f_1(x_2, t_0).$$

$$f_2(x_1, x_2, t) = f_1(\varphi^{(1)}_{t_0 - t}(x_1, x_2), t_0) f_1(\varphi^{(2)}_{t_0 - t}(x_1, x_2), t_0),$$

**Theorem. If** $\rho(x_1, x_2, t)$ **satisfies the Liouville equation then**

$f_1(x_1, t)$

**obeys to the following equation**

$$\left(\frac{\partial}{\partial t} + \frac{p_1}{m}\frac{\partial}{\partial q_1}\right) f_1(x_1, t) =$$

$$= \frac{1}{V}\int_{\Omega_V} \frac{\partial \Phi(|q_1 - q_2|)}{\partial q_1} \frac{\partial}{\partial p_1} [f_1(\varphi^{(1)}_{t_0 - t}(x_1, x_2), t_0) f_1(\varphi^{(2)}_{t_0 - t}(x_1, x_2), t_0)] dq_2\, dp_2.$$

**Bogolyubov type equation for two particles in finite volume**

- **Kinetic theory for two particles**

- **Hydrodynamics for two particles**

- **No classical determinism**
- **Classical randomness**

- **World is probabilistic (classical and quantum)**

**Compare: Bohr, Heisenberg, von Neumann, Einstein,…**

# Single particle (moon,…)

$$\rho = \rho(q, p, t)$$

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V}{\partial q}\frac{\partial \rho}{\partial p}$$

$$\rho\,\Big|_{t=0} = \frac{1}{\pi a b}\exp\{-\frac{(q-q_0)^2}{a^2} - \frac{(p-p_0)^2}{b^2}\}$$

- **Newton`s approach: Empty space (vacuum) and point particles.**
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# Fixed classical spacetime?

- **A fixed classical background spacetime does not exists (Kaluza—Klein, Strings, Branes).**

**There is a set of classical universes and a probability distribution** $\rho(M, g_{\mu\nu})$ **which satisfies the Liouville equation (not Wheeler—De Witt).**

**Stochastic inflation?**

# Functional General Relativity

- Fixed background $(M,g)$.

Geodesics in functional mechanics $\rho(x,p)$

Probability distributions of spacetimes $\rho(M,g)$

- No fixed classical background spacetime.
- No Penrose—Hawking singularity theorems
- Stochastic geometry?

# Example

$$\dot{x} = x^2,\ x(t) = \frac{x_0}{1 - x_0 t} \quad \text{singular}$$

$$\rho(x,t) = C\exp\{-(\frac{x}{1-xt} - q_0)^2/\varepsilon^2\},$$

$$\text{nonsingular}$$

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# Conclusions

**Functional formulation (non-Newtonian) of classical mechanics: distribution function instead of individual trajectories.**

**Fundamental equation: Liouville or FPK for <u>a single</u> particle.**

**Newton equation—approximate for average values.**
**Corrections to Newton`s trajectories.**

**Irreversibility.**

**<u>New kinetic theory, new hydrodynamics.</u>**

# Information Loss in Black Holes

- **Hawking paradox**.

- **Particular case of the Irreversibility problem.**

- **Bogolyubov method of derivation of kinetic equations -- to quantum gravity.**
- Th.M. Nieuwenhuizen, I.V. (2005)